OPTICAL LEVITATION

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ABSTRACT

Micron-sized drops of glycerol have been trapped and accelerated in a stable optical potential well provided by a focused beam from a continuous laser. The theory of how this works is presented in detail. An experiment is described that verifies the theory.

INTRODUCTION

The trapping and acceleration of micron-sized particles using the beam of a continuous laser was first reported by A. Ashkin (1) in 1970. Since that time, a number of papers have appeared on the subject.

The experiment described in this paper is similar to the Millikan Oil Drop experiment. In this case, instead of using an electric field to levitate an electrically charged drop as Millikan did, a radiation field, produced by a laser, is used to levitate an electrically neutral drop. This paper presents the theory of why a radiation field can be used to levitate a neutral drop and gives the details of an experiment that verifies it. This experiment is easily done where ever a 100 milliwatt laser is available.

THEORETICAL DEVELOPMENT

The geometry of the problem is shown in the ray diagram (Figure 1). The light ray is traveling normal to the earth's surface and is incident on a sphere that is partially transparent to that ray. The reflected and transmitted rays are shown. If the indicies of refraction

$$n(s) > n(m)$$

are known, the laws of reflection and refraction at the boundaries can be used to find the force imparted to the sphere by the incident light. This force (F_{rp}), which is due to the radiation pressure, must be greater or equal to the force of gravity (F_g) plus any viscous forces (F_v) acting on the sphere in order to trap or accelerate the sphere.

The force of gravity acting on the sphere is:

$$F_g = mg = 4/3\,\pi r^3 \rho g ,$$

where r is the radius of the sphere, ρ the density of the spherical oil drop, and g the acceleration due to gravity.

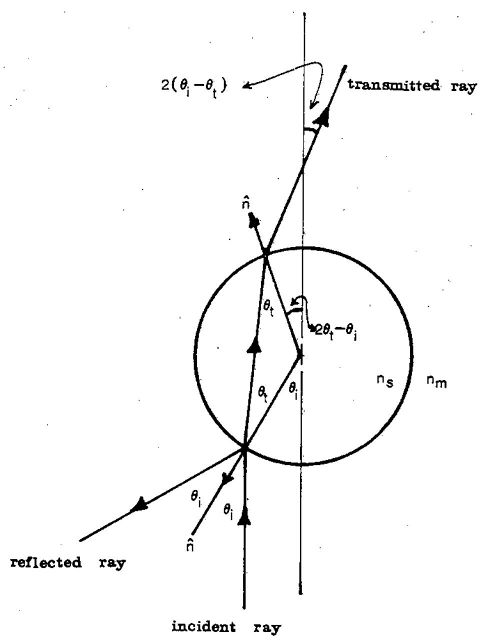

FIGURE 1
Ray diagram of the laser light as it passes through a spherical drop. The photons travel along the rays.

The viscous force acting will take on the form of Stokes Law:

$$\overline{F_v} = -6\pi r \eta \overline{v},$$

where η is the viscosity of the medium through which the drop is falling, and $\overline{v}$ is the velocity of the sphere. $\overline{F_{rp}}$ is equal to the sum of the vertical components of the forces due to the

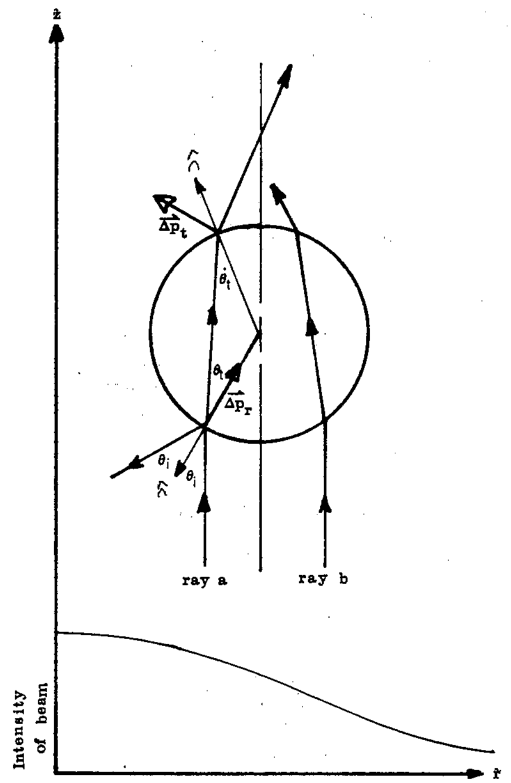

FIGURE 2
Momentum diagram of the light traveling through the drop. The drop is off the center line of the Gaussian distribution of the light intensity of the TEM_{00} mode of the Argon laser.

reflected and transmitted photons (the photon paths are represented by the rays).

These forces are shown in Figure 2 as change-in-momentum vectors. The two rays "a" and "b" are incident on a sphere that is off the center axis of the Gaussian distribution of the light intensity of the TEM_{00} mode of the laser. The net effect is that the hemisphere closer to the beam axis receives a horizontal force toward the beam center, while the other hemisphere receives a positive vertical force. Since the beam intensity (number of incident photons/area/time) decreases with distance away from the beam axis, the net horizontal force is always acting towards the beam center. This means that the drops will track along the beam axis.

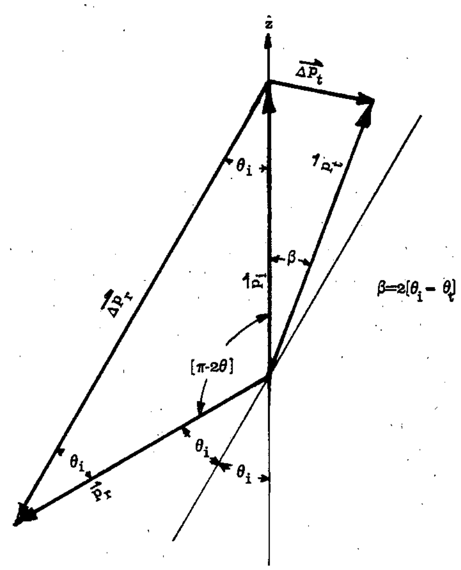

FIGURE 3
Vector momentum diagram for the laser light passing though the drop. These diagrams are used to find the z component of the force due to the radiation pressure.

Summing the z component of the forces due to the reflected photons (F_{rz}) and those due to the transmitted photons (F_{tz}) over the bottom half of the sphere gives:

$$\overline{F_{rp}} = \overline{F_{tz}} + \overline{F_{rz}} = \hat{z} N \left[(1-q) \int_0^{\pi/2} |\overline{\Delta P_{tz}}| \cos\theta_i \, d\beta + q \int_0^{\pi/2} |\overline{\Delta P_{rz}}| \cos\theta_i \, d\beta \right]$$

where N is the number of photons/area/time, q is the reflectance of the sphere, θ_i is the angle that the incident photon makes with the normal to the surface of the sphere $\hat{n}$, and $d\beta$ is the surface element. The incident momentum is given by:

$$\overline{P_i} = (h\nu/c)\hat{z}$$

where h is Planck's constant, ν the frequency, and c the speed of light. Using the ray diagram (Figure 1) and the vector diagram (Figure 3), the z

Transmitted	Reflected
$\overline{\Delta P_t} = \overline{P_t} - \overline{P_i}$	$\overline{\Delta P_r} = \overline{P_i} - \overline{P_r}$
to find z-component	to find z-component
$\lvert\overline{\Delta P_{tz}}\rvert = \lvert\overline{\Delta P_t}\rvert \sin(\theta_i - \theta_t)$	$\lvert\overline{\Delta P_{rz}}\rvert = \lvert\overline{\Delta P_r}\rvert \cos\theta_i$
$\lvert\overline{\Delta P_t}\rvert = 2\lvert\overline{P_i}\rvert \sin(\theta_i - \theta_t)$	$\lvert\overline{\Delta P_r}\rvert = 2\lvert\overline{P_i}\rvert \cos\theta_i$
$\lvert\overline{\Delta P_{tz}}\rvert = 2\lvert\overline{P_i}\rvert \sin(\theta_i - \theta_t)$	$\lvert\overline{\Delta P_{rz}}\rvert = 2\lvert\overline{P_i}\rvert \cos\theta_i$

component of the radiation pressure force becomes:

$$\overline{F^z_{rp}} = \hat{z}\, N \frac{h\nu}{c} 4\pi r^2 \left[(1-q)\int_0^{\pi/2} \sin^2(\theta_i - \theta_t)\sin\theta_i\cos\theta_i \, d\theta_i + q\int_0^{\pi/2} \cos^3\theta_i \, d\theta_i \right]$$

Snell's Law gives:

$$\theta_t = \arcsin\left[n(\text{medium})/n(\text{sphere}) \sin\theta_i \right]$$

The value of the reflectance can be measured by placing the oil in a glass container and shining the laser on it. The ratio of the power reflected to the power incident is the value of q. Using our measured value of q, the value of the quantity in brackets in equation 2 becomes 0.116.

The force due to the radiation pressure can be written as:

$$\overline{F^z_{rp}} = \left\{ N\,(h\nu/c)\; 4\pi r^2 Q \right\} \hat{z}$$

where Q is the value of the integrals in the square brackets. It should be noted that the effect of the internally reflected rays at the exiting boundary at the top of the drop are ignored. Their effect on the result is small.

If the beam has a cross-sectional area $= \pi\omega^2$ (which changes with the height because the beam is focused), the force can be written as:

$$\overline{F^z_{rp}} = 4PQr^2/c\omega^2\,\hat{z} = k/\omega^2\hat{z}$$

where ω is the radius of the beam and P is the total power in the incident beam.

In the region far below the focus waist (see Figure 4) at z=-2L, the velocity of the sphere is 0, so Newton's law tells us that the net force is zero. This means that

$$\overline{F^z_{rp}} = -\overline{F_g}$$

which becomes:

$$4PQr^2/c\omega^2 = 4/3\pi r^3 \rho g$$

Hence the radius of the sphere that would be trapped is given by:

$$r = 3PQ/(c\omega^2\pi\rho g)$$

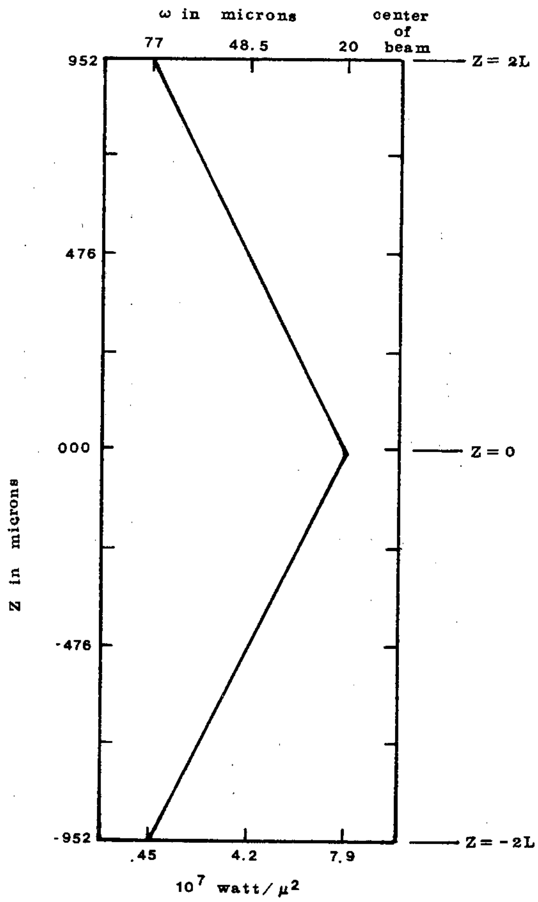

FIGURE 4
The geometry of the beam that was used to levitate the oil drops. The figure on the left is the power profile. The right figure is the spatial profile.

If the sphere obtains a small upward velocity, it will accelerate until it reaches z = -L/2 where terminal velocity is reached in accordance with Stoke's Law. To calculate the terminal velocity, one must remember that as the drop moves upward, the power/area changes. The size of the beam is given by:

$$\omega_p = sz + \omega$$

where s is the slope of the diverging beam (see figure 4), and ω the radius of the focus at the waist (z=0). The

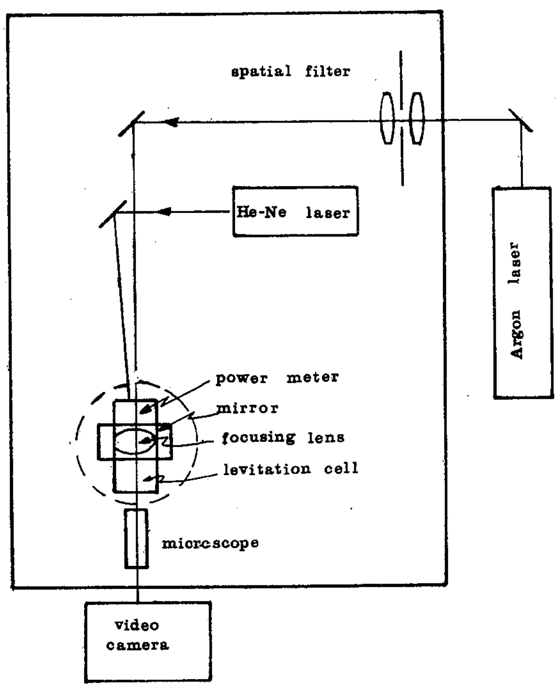

FIGURE 5
Experimental set-up for the optical levitation experiment.

velocity can be found using the energy balance integrals shown below. The energy put into the drop either goes into kinetic energy or gravitational energy.

$$\int_0^{L/2} \frac{k\ dz}{(sz+\omega_p)^2} = \int_0^{L/2} a\ v(z)\ dz + \int_0^{L/2} mg\ dz$$

where a is the coefficient in Stoke's Law.

The average velocity of the spheres in the central part of the focused beam (which can be measured) is given by:

$$\langle v\rangle = (1/L)\int_0^{L/2} v(z)\ dz$$

Performing these integrals gives the result:

$$\langle v\rangle = -\frac{8PQr^2}{Lca}\left[\frac{1}{s\left(\frac{sL}{2}-\omega_p\right)} - \frac{1}{s\omega_p}\right] - \frac{4\pi r^3 g\rho}{3a}$$

Everything in this equation is measurable. The aim of the experiment is to verify this result.

THE EXPERIMENT

A focused continuous laser beam (100 milliwatts power) is directed vertically into a glass cell in which glycerol drops may be sprayed from above. A properly calibrated viewing microscope is aligned horizontally to inspect the drops as they are trapped and accelerated near the focus of the beam. In the experiment reported here, the data were taken using a video-recorder as shown in Figure 5. Using the recorder allowed the data to be analyzed at leisure at a later time. A Helium-Neon laser was used to illuminate the calibration scale of the viewing microscope. This laser was used because its red color contrasted well with the blue light of the Argon laser used to levitate and illuminate the spheres. The brilliant scatter of the laser light off the drops can be seen easily by the unaided eye. The geometry of the beam (the slope s in equation 7) was measured by introducing smoke or water into the cell. The laser power was measured by the use of a power meter placed over the cell.

The average velocity was determined by measuring the time for the drop to move a distance of 476 microns. A total of 66 drops were tracked. The value of the slope of our beam was $(6.0\pm.5)\times10^{-3}$. The details of the laser beam profile are shown in Figure 4. Using oil of density (1256± 5)kg/m and reflectance q= 0.10±.01 and a 100 milliwatt laser determined that the radii of the trapped drops were r=.50 ±.05 microns. Together these gave a predicted average velocity of

$$\langle v\rangle\,(\text{theory}) = (3.0\pm.8)\times10^{-4}\ \text{m/sec}$$

This compares quite favorably with the measured value of:

$$\langle v\rangle\,(\text{experimental}) = (3.1\pm.4)\times10^{-4}\ \text{m/sec.}$$

CONCLUSION

The results obtained in this experiment support the theory developed as the experimentally measured velocity overlaps the theoretical value. The forces due to the focused laser light can trap and accelerate micron-sized electrically neutral semi-transparent particles. The only special piece of equipment needed to do this experiment is the 100 milliwatt laser. The rest of the material is commonly available in the advanced undergraduate laboratory.

ACKNOWLEDGMENTS

The author wishes to thank Professor Wangsness for his wonderful book on Electromagnetic Fields and Professor Wing for teaching the course in the manner he did. He is indebted to Professor Stegeman for providing the laboratory in which the experiment was done. Finally, he wishes to thank Professor K. C. Hsieh for his support.

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The Journal of
Undergraduate Research
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VOLUME 2, NUMBER 2

OCTOBER, 1983

Published by Guilford College
for
The American Institute of Physics and The Society of Physics Students