Terrestrial Imaging Through Atmospheric Turbulence

Jeff Thomson
OSSC
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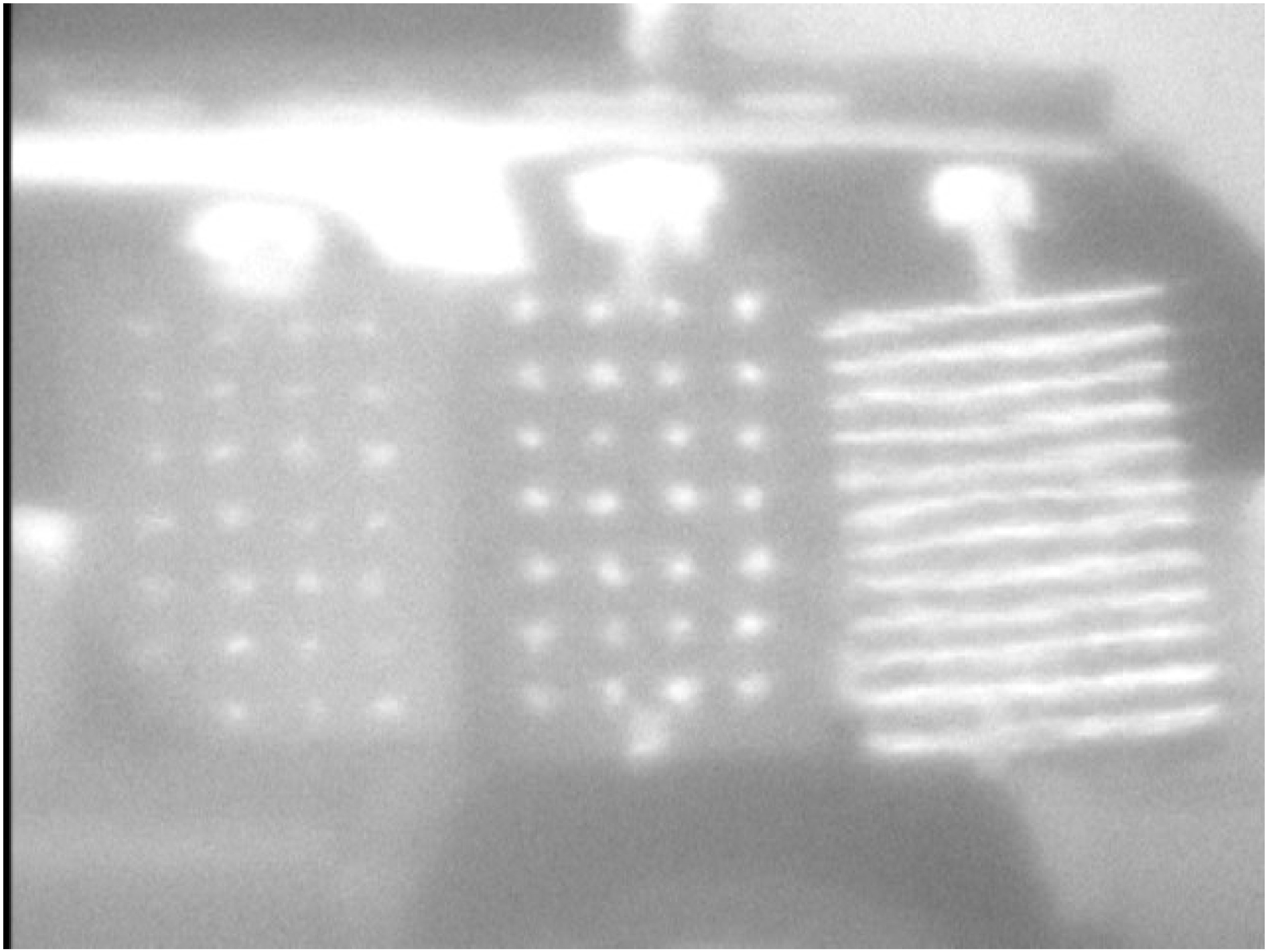
Imaging Through Moderate Turbulence



Terrestrial Imaging Through Turbulence - Applications

- Target identification
  - An example: people coming and going from an airport where close-in surveillance is not feasible
  - Sniper or drone attacks

- Long-range surveillance
  - Reading license plates
  - Identifying objects such as cars, contraband
- Periscope observation from submarines

Outline of Talk

- Differences between astronomical and terrestrial imaging
- Physics of atmospheric turbulence
  - C_n^2, r_o, t_o
- Effect of phase perturbations on imaging
  - $D/ r_{o,}$, short exposure vs long exposure
- Post-processing methods to obtain images
  - Lucky imaging
  - Deconvolution
- Hybrid methods
  - Wavefront sensor
  - Phase diversity
  - Deformable mirror
  - 3D imaging

Differences between Astronomical and Terrestrial Imaging

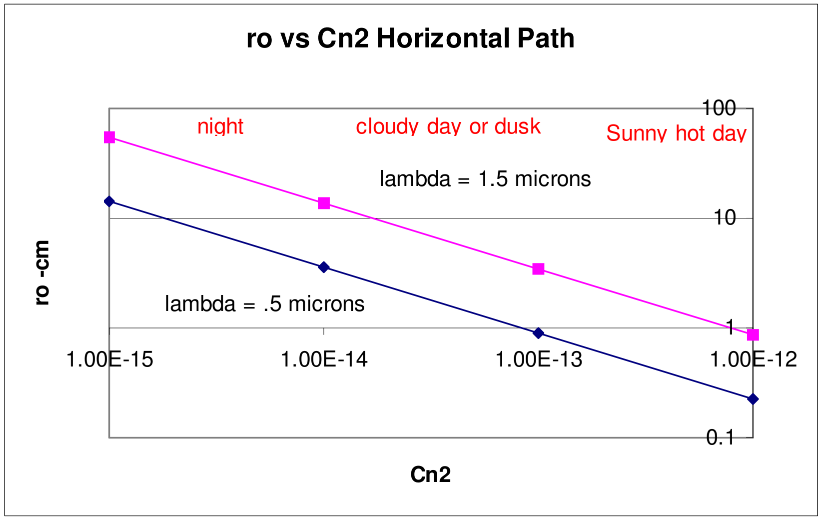
- Astronomical imaging
  - Atmosphere is comparatively thin layer near telescope
  - Targets are point sources within a small angular region
    - Generally within isoplanatic patch
- Terrestrial imaging
  - Atmosphere is approximately uniform between telescope and object
  - Images are extended, generally beyond isoplanatic patch
- Solar astronomy is a mixture
  - Atmosphere near telescope
  - Images extended over many isoplanatic patches

Physics of Atmospheric Turbulence

- Temperature fluctuations are driven by thermal difference between surface and air
  - Turbulent flow conditions create cascade of energy from “outer scale” L_o to “inner scale” l_o, where turbules dissipate due to molecular diffusion
- Kolmogorov theory
  - Structure function: $D(r) = C_T^2\, r^{2/3}$
  - Power spectral density: $\Phi_T(\kappa) = .033\, C_T^2\, \kappa^{-11/3}$
- Temperature fluctuations lead to density fluctuations
- Density fluctuations lead to phase fluctuations
  - Structure constant: $\Phi_N(\kappa) = .033\, C_N^2\, \kappa^{-11/3}$

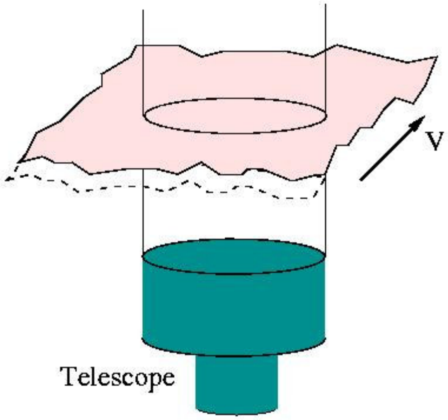
Turbulence Effects on Optical Propagation

- Localized density variations (turbules) act as weak random lenses
  - Small change in index of refraction (~10^{-6}) can add up over distance, e.g. star image:
    - Intensity scintillations
    - Beam wander
    - Apparent spreading



- Fried coherence diameter

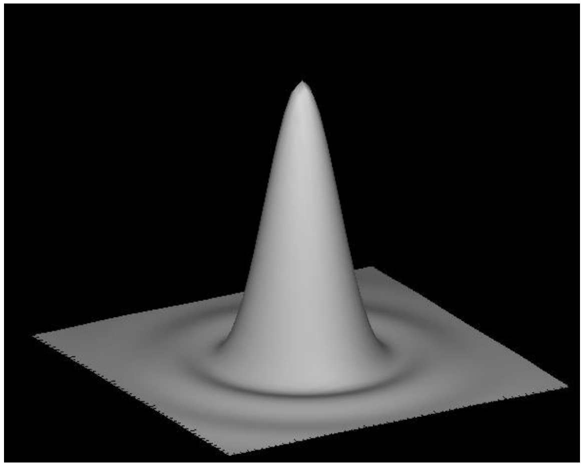
$$r_o = \left[.432k^2 \sec(\theta) \int_0^L C_n^2(z) \left(\frac{z}{L} \right)^{5/3} dz \right]^{-3/5} \approx 3\left(C_n^2 L k^2\right)^{-3/5}$$

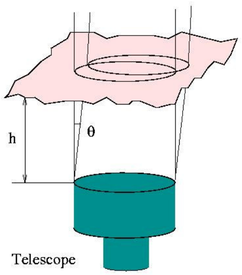


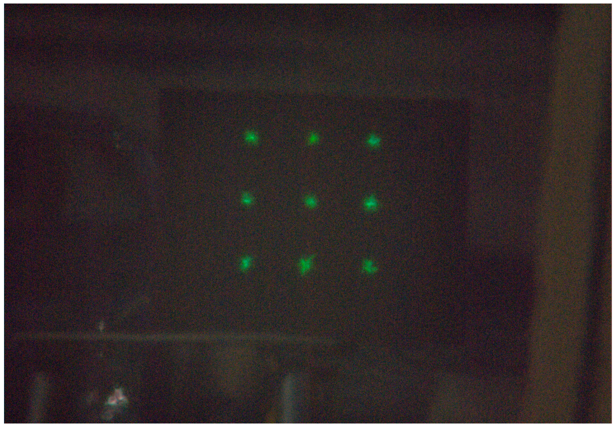
- Coherence time $t_o = r_o / V$

Imaging – PSF, Anisoplanicity

- Point spread function is a measure of the quality of image
  - Diffraction limited image: PSF FWHM ~ 1/D, peak intensity ~ $1/D^2$
  - Strehl Ratio (S) is ratio of actual peak intensity to diffraction-limited
  - In presence of atmospheric turbulence S is reduced
    - No turbulence: S =1
    - Long exposure: $S \sim (D/r_o)^{-2}$
    - Short exposure : $S \sim (D/3.4r_o)^{-2}$
- Anisoplanicity – separated parts of image pass through non-correlated paths
  - Strehl only same in isoplanatic patches
  - Isoplanatic angle ~ r_o/L





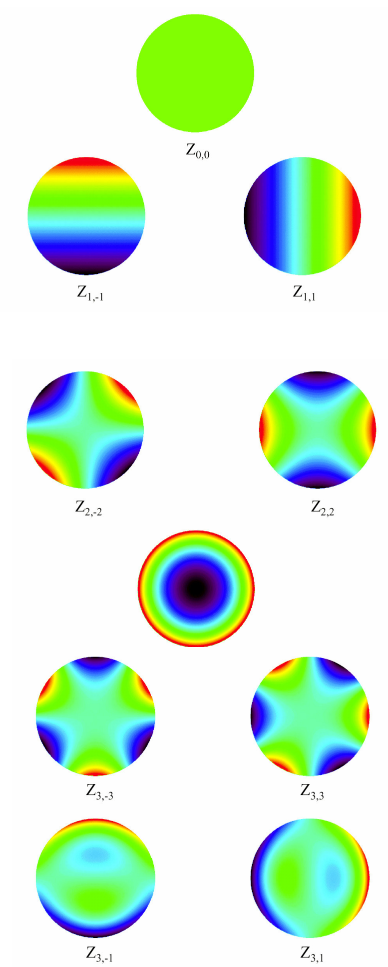


Imaging – Zernicke Polynomials

- Phase aberrations cause deformed wavefront
  - Convenient to describe in terms of orthonomial polynomials on unit circle
    - Zernike polynomials popular because they have familiar optical interpretation: tip/tilt, focus, coma, etc.

$$\Phi(r,\theta) = \sum_{2}^{\infty} a_n Z_n(r,\theta)$$

    - Karhunen-Loeve polynomials: coefficients are statistically independent



Atmospheric Effects on Phase Perturbations

- Phase structure function

$$D(r) = 2\left[\left\langle \phi(r_1)^2 \right\rangle - \left\langle \phi(r_1)\phi(r_1 + r)\right\rangle\right] = 6.88(r / r_0)^{5/3}$$

- Spatial spectrum can be used to evaluate average phase over aperture: $<\Delta\phi^2>$
  - Strehl ~ Exp(- $<\Delta\phi^2>$)=Exp(-$\Sigma\Delta_n$ $(D/ro)^{5/3}$)
  - Good viewing: $<\Delta\phi^2> < 1$

- No mode correction corresponds to long exposure:

$$<\Delta\phi^2> = 1.0299\ (D/r_o)^{5/3}$$

- Short exposure corresponds to removal of tip/tilt:

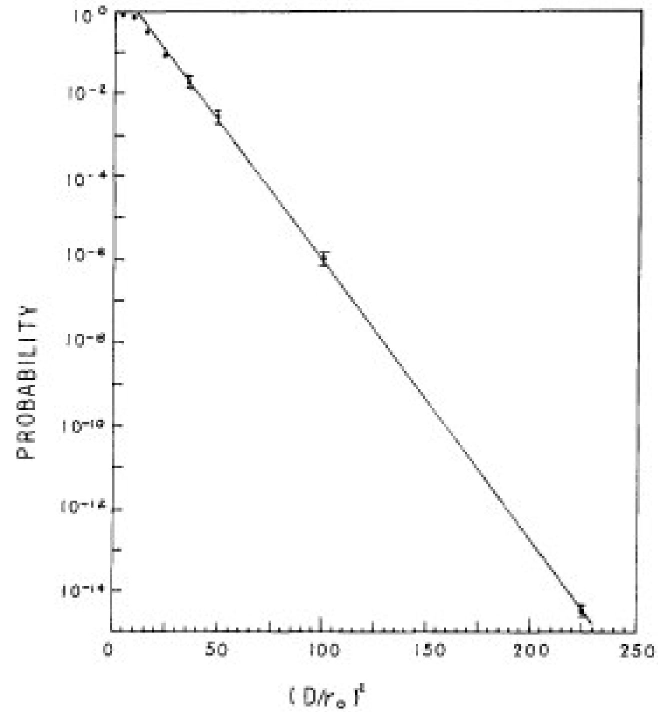
$$<\Delta\phi^2> = .134\ (D/r_o)^{5/3} = (D/3.4r_o)^{5/3}$$

Methods of Improving Images

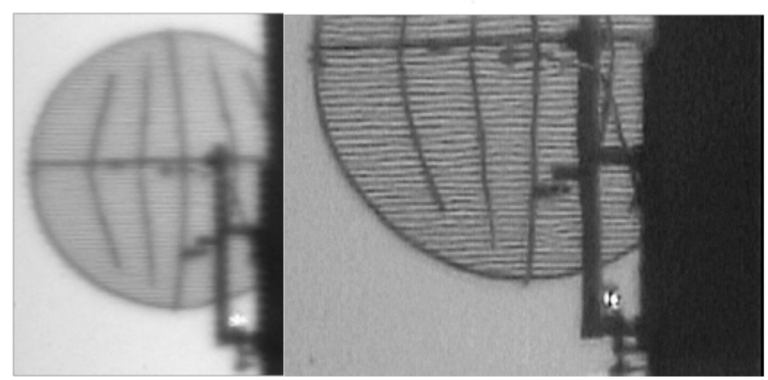
- Post Processing Methods
  - Lucky Viewing
  - Deconvolution
- Hybrid (processing + extra hardware)
  - Deconvolution with wave front sensor
  - Phase Diversity with 2 cameras
  - Phase Diversity with deformable mirror
  - 3-D imaging

Lucky Imaging

- Since distribution of lenses is random, there could be a situation where $\Delta\phi^2 < 1$, if you are lucky
  - Dave Fried in1978 calculated odds of this happening
  - The a_n coefficients of Z_n are random Gaussian variables
  - a_n^2 is chi-square distributed of order one
  - A Monte Carlo calculation resulted in this graph



- If enough images are taken (depending on D/ro), the lucky isoplanatic patches can be identified and stitched together
  - Patches are aligned according to long exposure image
  - This image taken at 1 km over local ocean
  - Infer D/ro ~ 4 - 5



Deconvolution

- Image is convolution of telescope and atmosphere PSF

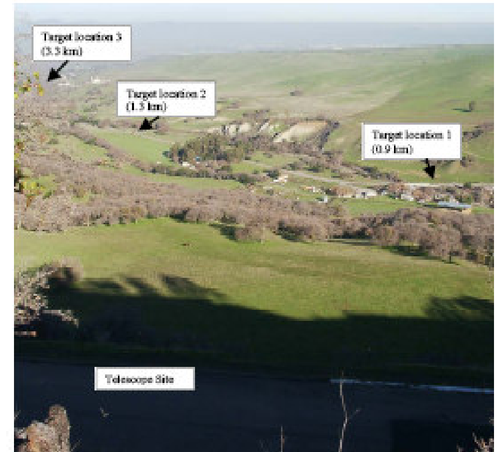
$$i_n(\vec{x}) = \tau_n(\vec{x}) * o(\vec{x})$$

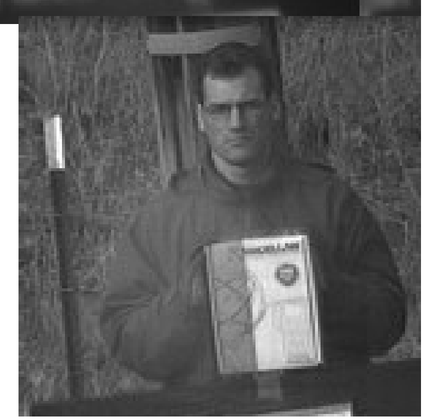
- In Fourier space, becomes multiplication
- Fourier magnitude: $$|O(\mathbf{u})|_{est} = \left[\frac{\left\langle |I_n(\mathbf{u})|^2 \right\rangle_n}{\left\langle |\tau_n(\mathbf{u})|^2 \right\rangle_n} \right]^{1/2}$$

- Fourier phase estimated using bispectrum:

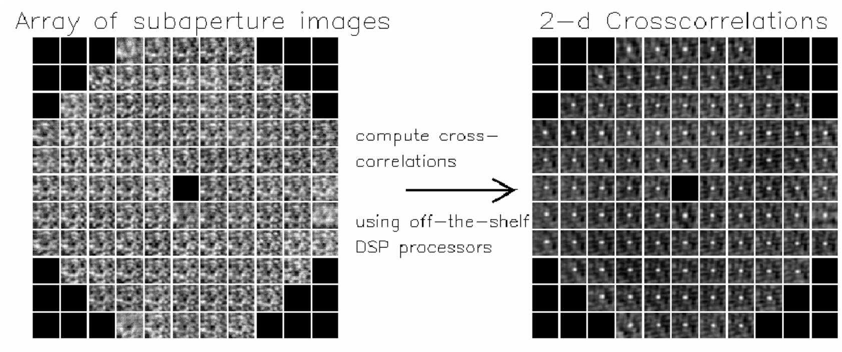
$$I_{B,n}(\mathbf{u},\mathbf{v}) \equiv I_n(\mathbf{u}) I_n(\mathbf{v}) I_n(-\mathbf{u}-\mathbf{v}),$$

$$\arg|O(\mathbf{u}+\mathbf{v})| = \arg|O(\mathbf{u})| + \arg|O(\mathbf{v})| - \arg\left|\left\langle I_{B,n}(\mathbf{u},\mathbf{v}) \right\rangle_n\right|$$



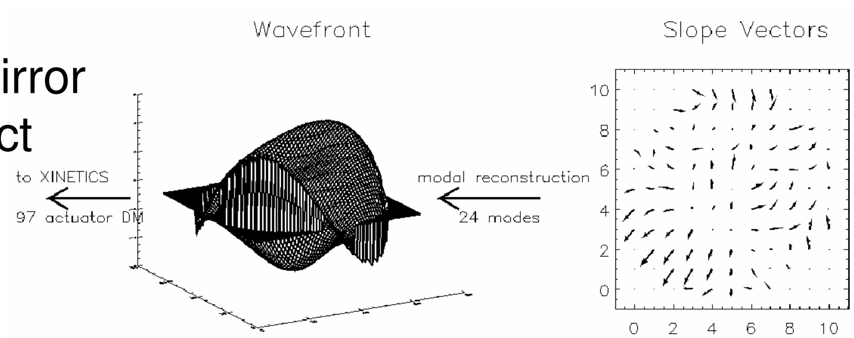


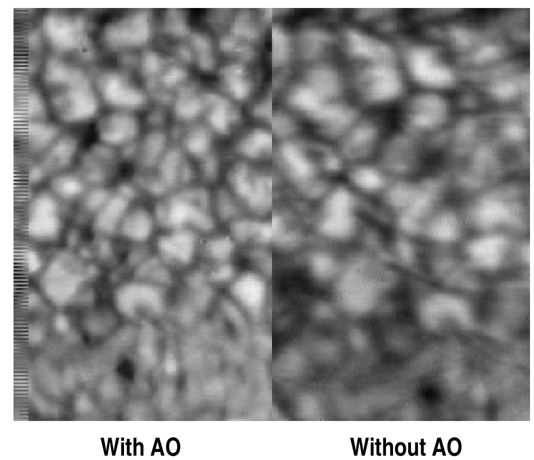
Low Order AO & Deconvolution with Wave Front Sensor



1) Spatial correlation of images in WFS allows estimate of aberrated pupil function

Feedback to deformable mirror gives AO effect





2) Estimate of pupil function allows better estimator to be used in deconvolution calculation

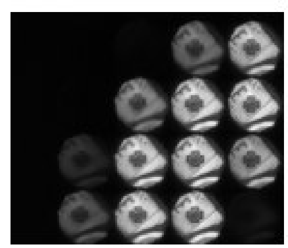


Figure 2. Portion of WFS CCD image from experiment. The shamrock is successfully used for scene-based wavefront sensing.

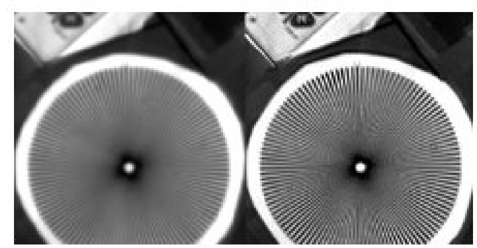


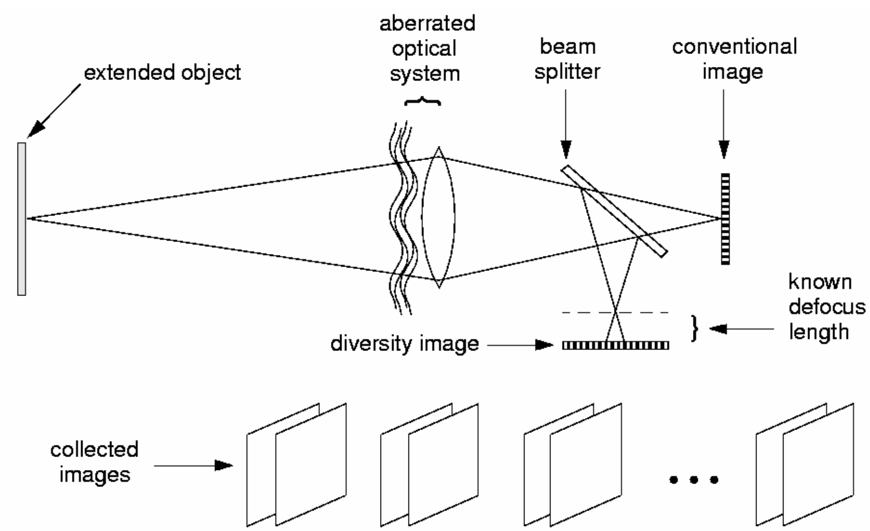
Figure 3. Left: shift-and-add image of resolution target near the shamrock. Right: image deconvolved with scene-based WFS data. This technique improves target contrast and resolution.

Phase Diversity

- Method uses 2 images taken with known separation
  - Phase fronts can be measured using wavefront curvature algorithm:

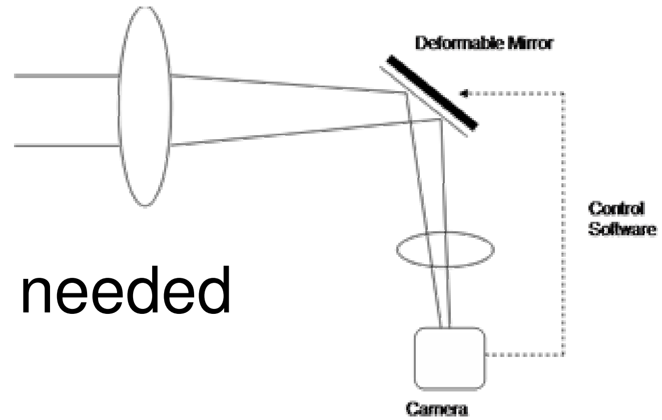
$$\frac{\partial I}{\partial z} = \frac{I_f - I_{df}}{\nabla z} \qquad \nabla^2 \phi = -\frac{k}{I}\frac{\partial I}{\partial z}$$

  - Used to obtain wavefront deconvolution or as input to DM

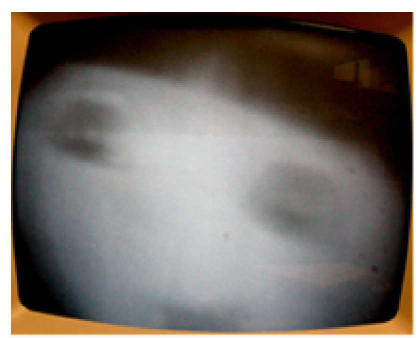


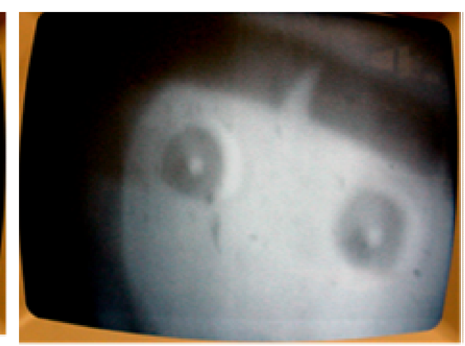
- Multiple images needed

Phase Diversity with Deformable Mirror

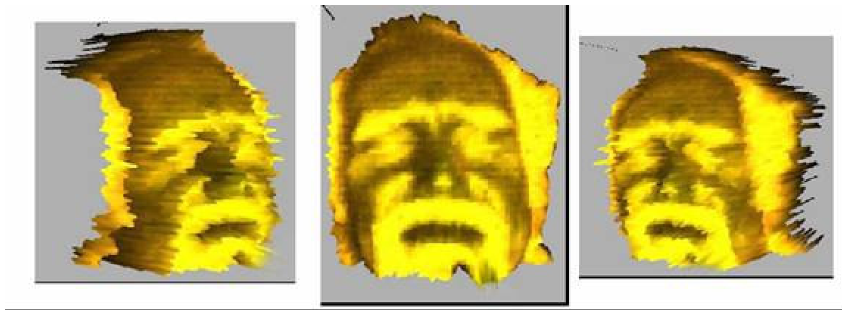


- Addition of deformable mirror allows multiple known aberrations to be induced
  - Only one beam path and camera needed
- Making your own luck:
  - Regions of the image can be greatly improved with heuristic control of few DM actuators
  - Within correlation time, can create the effect of multiple lucky regions
  - Need fast DM





3D Imaging



- An innovative 3D imager has recently been developed by Advanced Scientific Concepts
  - Illuminating laser pulse is emitted, returned light is imaged at 30 Hz – short exposure time
  - Sensor array measures time of flight potentially to mm precision: $\delta l_r = 30 / SNR$ (mm)
- Turbulence effects:
  - Range precision: $\delta l_r = .176\ \lambda\ (L_o/r_o)^{5/6}$
  - Cross-range resolution: $\delta l_{xr} = \lambda\ L/(3.4\ r_o)$
- Required resolution unknown
- Synergy between image reconstruction techniques and range information unknown (if any)
- Laser illumination of target allows operation in low turbulence regimes, e.g. at night or dusk